New Concepts in Gravitation

Pari Spolter

Abstract

The gravitational force of the Sun, based on observations, is derived as the product of the acceleration times the area of a circle with radius equal to the semimajor axis of revolution. This quantity is constant for all planets, asteroids, and artificial satellites; it is independent of the mass of the attracted body. The equation for the sequential mean distance of the planets from the center of the Sun is derived as $r = B \cdot C^n$*, where B and C are constants and n is the sequential number of the bodies. The correlation coefficient is 0.997. It is concluded that gravitation is quantized. When the gravitational force is calculated by this new equation (*$\boldsymbol{F}_S = \boldsymbol{a} \cdot A$*), there is a highly significant correlation between the magnitude of perturbative forces and the eccentricity of the orbit of the planets and the asteroids. A graph of the maximum inclination of the orbit of each planet to the equatorial plane of the Sun shows no correlation between the inclination and the eccentricity of the orbit. Thus general relativity cannot explain the eccentricities. The residual advance of the perihelion of Mercury of about 0.1″ per revolution is explained by the fact that the direction of the advance coincides with the direction of the movement of the solar system in space, as detected recently by measurements of anisotropy in the cosmic microwave background radiation. An equation for the eccentricity is presented as the ratio of the sum of perturbations to the gravitational force of the Sun. By analysis of data it is shown that Kepler's second law is not a general law; i.e., equal areas are swept in approximately equal intervals of time only near the aphelion and the perihelion. Indeed, if Kepler's second law were a general law, it would be inconsistent with his first and third laws. New units of force and energy are presented.*

Key words: gravitation, Kepler's third law, Titius–Bode law, distance law, perihelion advance of Mercury, eccentricity, Kepler's second law, units of force and energy.

1. INTERPRETATION OF KEPLER'S THIRD LAW

The Danish astronomer Tycho Brahe (1546–1601) had accumulated data from 20 years of very accurate pretelescopic observations of the positions and motions of the six planets known at that time. After Tycho's death, his assistant Johannes Kepler came into possession of Tycho's records. On the evening of 8 March 1618, the 46-year-old Kepler, who had gone over Tycho's documents for 17 years, deciphered the constancy of the ratio of the cube of the mean distance of each planet from the Sun (r) to the square of its sidereal period of revolution (t). For all the six planets, the ratio r^3/t^2 was the same number! This relationship is the well-known Kepler's third law of planetary motion, which was published in *Harmonices Mundi* (*The World Harmony*) in 1619. Later, when the other three planets were discovered and the orbits of more than 4000 asteroids were determined, again the ratio of r^3/t^2 was the same number. This constant ratio was also noted when artificial satellites were placed in heliocentric orbits in the 1960s.

A plot of the orbital velocity, v, of the planets at semimajor versus the semimajor axis of revolution, r, is shown in Fig. 1. These same data, plotted on logarithmic paper, are presented in Fig. 2 together with the least squares line of regression. The equation for this line, with the intervals calculated at the 95% confidence level, is

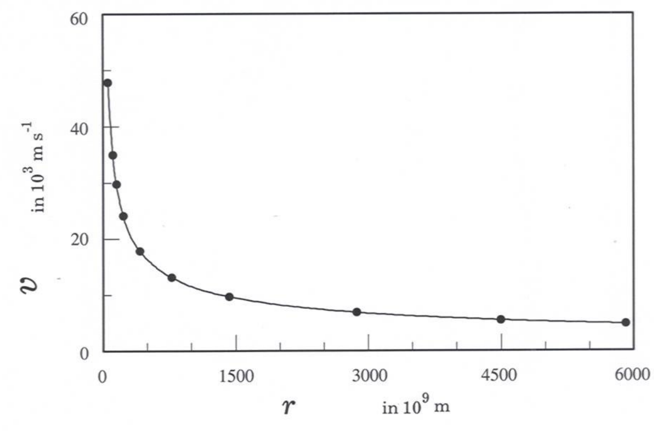

Figure 1. Mean orbital velocities of the planets plotted as a function of their mean distances from the Sun.

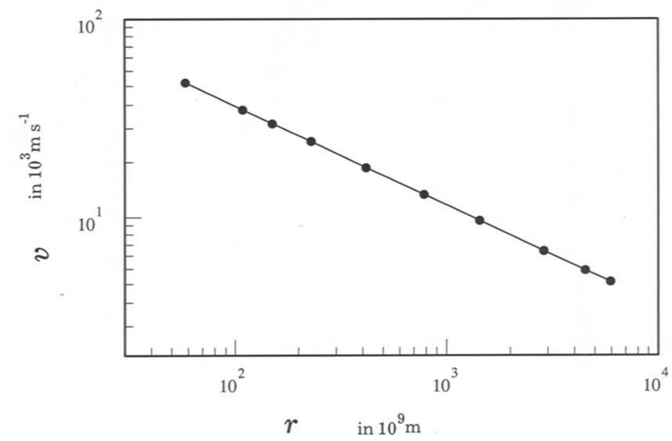

Figure 2. Same data as in Fig. 1, plotted on logarithmic paper. The line drawn is the least squares regression line.

$$v = (364.0877 \pm 0.0463) \cdot r^{(-0.500\,007 \pm 0.000\,0078)}. \qquad (1)$$

The correlation coefficient is 0.9999.

The intervals for the intercept are calculated at the arithmetic means.

Squaring both sides and rearranging, we get

$$\begin{aligned} &v^2 \cdot r = 132\,559.8448 \times 10^6 \times 10^9 \\ &\text{or} \\ &1.325\,598\,448 \times 10^{20}\ \text{m}^2/\text{s}^2 \cdot \text{m}. \end{aligned} \qquad (2)$$

We can divide and multiply the left side of the equation by r without changing the equation. We can also multiply both sides of this equation by a constant, π. We get

$$\frac{v^2}{r} \cdot \pi r^2 = 4.164\,49 \times 10^{20}\ \text{m}/\text{s}^2 \cdot \text{m}^2. \qquad (3)$$

What is constant for all the planets is the gravitational force of the Sun:

$$\mathbf{F}_S = \frac{v^2}{r} \cdot \pi r^2$$

or

$$\begin{aligned} \mathbf{F}_S &= \text{acceleration} \times \text{area} \\ &= \mathbf{a} \cdot A. \end{aligned} \qquad (4)^1$$

This, of course, is Kepler's 385-year-old third law of planetary motion, shorn of its mystery. (If we replace v with $2\pi r/t$, this equation becomes r^3/t^2 = constant.)

The value of the gravitational force of the Sun, calculated for each planet, using this equation, is shown in Table I.

In the 1960s several artificial satellites were placed in heliocentric orbits, by NASA and by the former Soviet Union. Using (4), we can calculate the gravitational force of the Sun from the orbit of each satellite. The data for some of these satellites are summarized in Table II.

2. IS F = *ma*?

Let us now calculate the gravitational force of the Sun using Newton's formula

$$\mathbf{F} = m \cdot \mathbf{a} = m \cdot \frac{v^2}{r}. \qquad (5)$$

Table III shows the data for the planets. The figures used for v and r of each planet are those given in Table I.

The gravitational force of the Sun, calculated from the orbit and the mass of each artificial satellite, using Newton's equation (5), is shown in Table IV. The figures used for v and r of each satellite are those given in Table II.

If we use Newton's universal law

$$F = \frac{GMm}{r^2}, \qquad (6)$$

Table I: Gravitational Force of the Sun, Calculated from the Orbit of Each Planet Using (4)

Number n	Planet	v Orbital velocity semimajor[a] $\times 10^3$ m/s	r Semimajor axis of revolution[a] $\times 10^9$ m	$\mathbf{F}_S = \mathbf{a}A$ $\times 10^{20}$ m/s$^2 \cdot$ m^2
1	Mercury	47.828	57.95	4.1645
2	Venus	35.017	108.11	4.1646
3	Earth	29.771	149.57	4.1646
4	Mars	24.121	227.84	4.1646
5	Asteroids[b]	17.892	414.1	4.1646
6	Jupiter	13.052	778.14	4.1645
7	Saturn	9.6383	1427.0	4.1646
8	Uranus	6.7951	2870.3	4.1636
9	Neptune	5.4276	4499.9	4.1645
10	Pluto	4.7365	5909	4.1646

[a] Values from *CRC Handbook of Chemistry and Physics,* 64th edition, 1983–1984, pp. F–130 and F–133.
[b] Values for the two largest asteroids Ceres and Pallas.

Table II: Gravitational Force of the Sun, Calculated from the Orbit of Each Artificial Satellite, Using (4)

Name		Launch Date	v Mean Orbital Velocity 10^3 m/s	r Mean Distance $\times 10^9$ m	$\mathbf{F}_S = \mathbf{a}A$ $\times 10^{20}$ m/s$^2 \cdot$ m^2
Luna 1[a]	USSR	2 Jan 59	27.80	172.03	4.17
Pioneer 5[b]	NASA	11 Mar 60	31.40	134.54	4.16
Mariner 2[b]	NASA	27 Aug 62	30.22	144.63	4.15
Ranger 5[b]	NASA	18 Oct 62	29.74	149.67	4.16
Mars 1[b]	USSR	1 Nov 62	26.49	189.07	4.16
Mariner 4[b]	NASA	28 Nov 64	25.72	200.60	4.16
Pioneer 6[b]	NASA	16 Dec 65	31.43	134.56	4.17
Pioneer 7[b]	NASA	17 Aug 66	28.82	159.69	4.16
Mariner 5[b]	NASA	14 Jun 67	36.73	98.28	4.16
Mariner 6[b]	NASA	24 Feb 69	26.23	192.83	4.16
Mariner 7[b]	NASA	27 Mar 69	26.44	189.91	4.17
Mars 4[a]	USSR	21 Jul 73	26.27	191.48	4.15

[a] Calculated from data in Charles S. Sheldon II, "Table of Soviet Space Launches, 1957–1975," in *Soviet Space Programs, 1971–75*, Vol. 1 (U.S. Government Printing Office, Washington, DC, 1976), pp. 553–608.
[b] Calculated from data in *Space Log*, Vol. 18, edited by J.M. Mathews (TRW Inc., Redondo Beach, CA, 1981), pp. 14–95.

and the currently accepted values for G (6.67259×10^{-11} N $\cdot$ m^2/kg^2), for the mass of the Sun (1.9891×10^{30} kg), and for the mass of each planet or artificial satellite (given in Tables III and IV) and for the distances as given in Tables I and II, we obtain the same figures for **F** as those listed in the last columns of Tables III and IV. Thus, if we accept Newton's force laws, we have to assume that the Sun, somehow, recognizes each body around it and doles out a specific amount of its attractive force depending on the particular body that orbits it. The gravitational force of the Sun calculated from Newton's equations is not constant and varies from 4.16×10^{23} N for Jupiter to only 0.31 N for Pioneer 5, with a different value for each of the other bodies listed in the two tables.

3. THE DISTANCE LAW

We saw in the preceding section that the product $v^2 r$ is a constant for all the planets. The next question is, can a planet be at any distance (r) from the Sun, so long as this constant product is obtained? In other words, are planets at random distances from the Sun, or does the sequence obey a mathematical law?

A mathematician looking at columns 1 and 4 of Table I may suspect some kind of progression. To explain the pattern mathematically Johann Daniel Titius (1729–1796), a German astronomer and physicist,

Table III: Gravitational Force of the Sun, Calculated from the Orbit and the Mass of Each Planet Using Newton's Equation (5)

Number *n*	*Planet*	*Mass*[a] $\times 10^{24}$ *kg*	***F** = m**a**?* *kg · m/s*^{2} *(newtons)*
1	Mercury	0.33022	1.30×10^{22}
2	Venus	4.8690	5.52×10^{22}
3	Earth	5.9742	3.54×10^{22}
4	Mars	0.64191	1.64×10^{21}
5	Asteroids:		
	Ceres	0.0009945	7.69×10^{17}
	Pallas	0.0002785	2.15×10^{17}
6	Jupiter	1898.8	4.16×10^{23}
7	Saturn	568.50	3.70×10^{22}
8	Uranus	86.625	1.39×10^{21}
9	Neptune	102.78	6.73×10^{20}
10	Pluto	0.015	5.69×10^{16}

[a] Values from *The Astronomical Almanac for the Year 1993* (U.S. Government Printing Office, Washington, DC, 1993), p. E88, for the planets, and adapted from E. Myles Standish Jr. and Ronald W. Hellings, *Icarus* **80,** 326 (1989) for the two asteroids Ceres and Pallas.

found in 1766, by trial and error, a purely *ad hoc* rule that fit the progression. In 1772 Johann Elert Bode (1747–1826), a German astronomer, incorporated this formula in the second edition of his introductory astronomy book.

The law is obtained by writing down first 0, then 3, and then doubling the previous number: 6, 12, 24, …. If 4 is added to each number and the sum divided by 10, the resulting numbers would give the mean distances of the orbits of the planets in astronomical units (see Table V).

When Uranus was discovered by William Herschel in 1781, its mean orbital distance from the Sun approximately fit the Titius–Bode law. This generated confidence in the law and drew attention to the gap between the orbits of Mars and Jupiter. A concerted effort was made by many astronomers to search for the missing planet. Rather than one large planet, a number of small bodies were found at a mean distance close to that predicted by the Titius–Bode law. The first and largest asteroid, Ceres, was discovered by Giuseppe Piazzi (1746–1826) in 1801. The rule breaks down completely for Neptune and Pluto, which were discovered later.

Contemporary astronomy textbooks dismiss the Titius–Bode law as merely a numerical coincidence and a historical curiosity, without physical basis.

A plot of the mean distance of the planets from the Sun, r, versus the sequential numbers, n, is shown in

Table IV: Gravitational Force of the Sun, Calculated from the Orbit and the Mass of Each Artificial Satellite Using Newton's Equation (5)

Name		*Launch date*	*Mass kg*	***F** = m**a**?* *kg · m/s*^{2} *(newtons)*
Luna 1[a]	USSR	2 Jan 59	361	1.62
Pioneer 5[b]	NASA	11 Mar 60	43	0.31
Mariner 2[b]	NASA	27 Aug 62	203	1.28
Ranger 5[b]	NASA	18 Oct 62	343	2.03
Mars 1[b]	USSR	1 Nov 62	893.5	3.32
Mariner 4[b]	NASA	28 Nov 64	261	0.86
Pioneer 6[b]	NASA	16 Dec 65	63.5	0.47
Pioneer 7[b]	NASA	17 Aug 66	63.5	0.33
Mariner 5[b]	NASA	14 Jun 67	245	3.36
Mariner 6[b]	NASA	24 Feb 69	412.8	1.47
Mariner 7[b]	NASA	27 Mar 69	412.8	1.52
Mars 4[a]	USSR	21 Jul 73	4650	16.76

[a] Data for the weight of the satellites reported in Charles S. Sheldon II, in *Soviet Space Programs, 1971–75*, Vol. 1 (U.S. Government Printing Office, Washington, DC, 1976), pp. 553–608.

[b] Data for the weight of the satellites reported in *Space Log*, Vol. 18, edited by J.M. Mathews (TRW Inc., Redondo Beach, CA, 1981), pp. 14–95.

Table V: Mean Distance of the Planets from the Sun

Planet	*Calculated According to Titius–Bode Law (in AU)*	*Observed (in AU)*
Mercury	0.4	0.39
Venus	0.7	0.72
Earth	1.0	1.0
Mars	1.6	1.52
Asteroids	2.8	2.77[a]
Jupiter	5.2	5.20
Saturn	10.0	9.53
Uranus	19.6	19.2
Neptune	38.8	30.1
Pluto	77.2	39.8

[a] Values for the two largest asteroids Ceres and Pallas.

Fig. 3. The figures used for n and r are those given in columns 1 and 4 of Table I.

The same data, plotted on semilogarithmic paper, are presented in Fig. 4 together with the least squares line of regression.

The equation for this line, with the intervals calculated at the 95% confidence level, is

$$r = \left(31.946 \begin{matrix} +3.4 \\ -3.1 \end{matrix} \right) \times (1.71 \pm 0.06)^n \text{ in } 10^9 \text{ m.} \qquad (7)$$

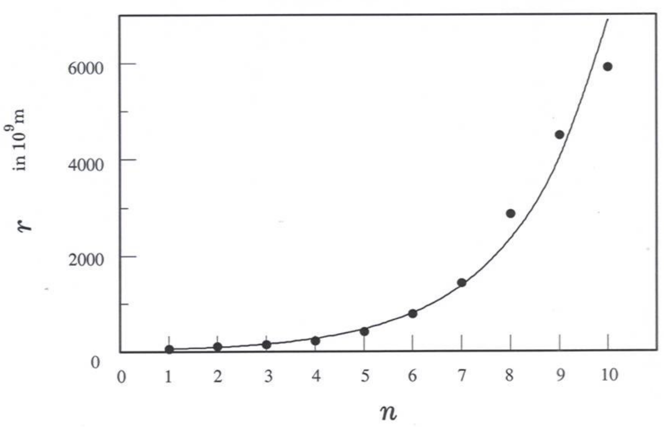

Figure 3. Mean distances of the planets from the Sun, *r*, plotted as a function of the sequential numbers, *n*.

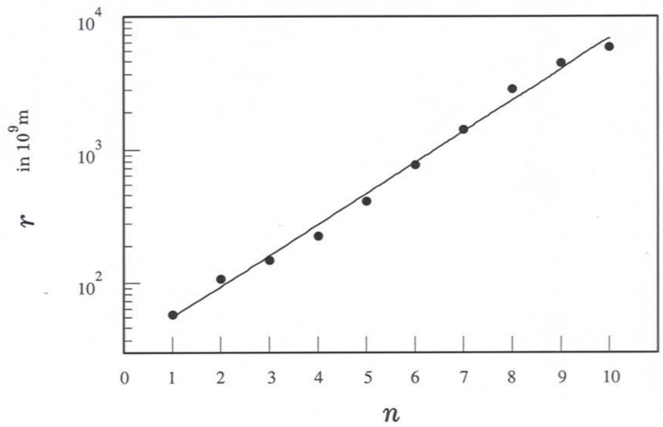

Figure 4. Same data as in Fig. 3, plotted on semilogarithmic paper. The line drawn is the least squares regression line.

The correlation coefficient is 0.9967.

We conclude that the distance law is an integral part of gravitation; i.e., gravitation is quantized.

4. PERIHELION ROTATION

All planets move in elliptical orbits around the Sun. The orbit of some planets, such as Venus ($e = 0.0067$) and Earth ($e = 0.0167$), is nearly circular. Other planets, such as Mercury ($e = 0.2056$) and Pluto ($e = 0.250$), have orbits of high eccentricity. The orbital elements of each planet change with time due to perturbations by the other planets. Analytical methods in classical celestial mechanics were developed by Leonhard Euler (1707–1783), by Joseph Louis Lagrange (1736–1813), and by Pierre Simon Laplace (1749–1827) in the 18th century to calculate the theoretical changes in the orbital elements of the planets due to their mutual attractions on each other. In most cases the predictions agreed with the observed motion of the planets, but there were a few exceptions where a small discordance was noted.

In 1859 the French mathematician and astronomer Urbain Jean Joseph Le Verrier (1811–1877) reported that 38″ of the 5600″ of the observed advance in the perihelion of Mercury per century could not be accounted for by adding together the general precession and the gravitational effects of the known planets.[1]

In 1895 Simon Newcomb (1835–1909), the leading astronomer and superintendent of the American Nautical Almanac, published *The Elements of the Four Inner Planets and the Fundamental Constants of Astronomy*.[2] He concluded that the observed motion of the perihelion of Mercury was 41″ per century greater than the expected theoretical value.

In 1915 Einstein expanded the space-time concept of the special theory of relativity to explain gravitation. According to this new theory, called general relativity, gravitation is a consequence of the geometric properties of the four-dimensional space-time continuum. Space-time is curved in the vicinity of matter. The "curved space" determines the motion of bodies in the field. The concept of a centripetal force is completely eliminated. In another paper published in 1915[3] Einstein claimed that his new theory of gravitation explained the anomalous motion of the perihelion of Mercury.

Figure 5 shows the orbit of Mercury in a warped space as visualized in a three-dimensional model. The warping of the space-time continuum would, according to the general theory of relativity, cause a rotation of the perihelion of the planets.

In deriving the field equations of general relativity, Einstein has assumed spherical symmetry; i.e., the space-time continuum is curved in a uniform manner around a celestial body, as shown in Fig. 5 in a three-dimensional model. Then why are the orbits of all the planets not perfect circles? What causes the eccentricities?

The inclination of the planet's orbit cannot explain the eccentricities. Mercury's orbit is inclined 3.39° to the solar equatorial plane and has an eccentricity of 0.2056; whereas the Earth's orbit is inclined 7.25° to the solar equatorial plane, and its eccentricity is only 0.0167. The inclinations of the orbits of the planets to the Sun's equatorial plane are given in Table VI.

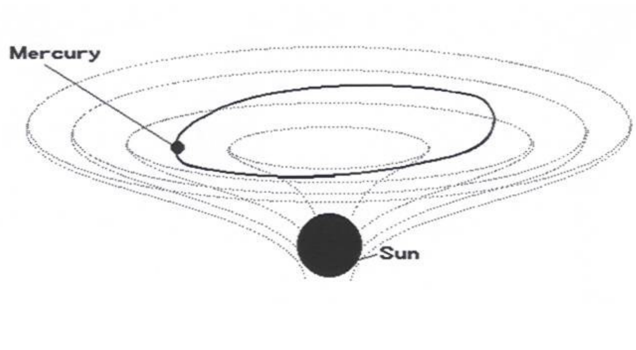

Figure 5. Orbit of Mercury in a warped space, according to the general theory of relativity, visualized in a three-dimensional model.

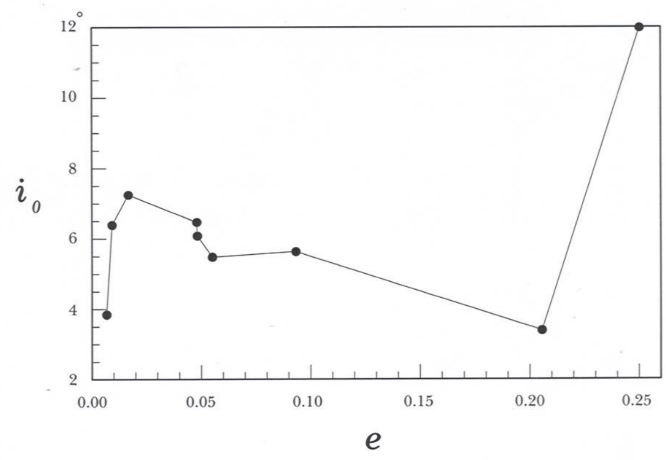

Figure 6. The maximum inclination of the orbit of the planets to the equatorial plane of the Sun, i_0, plotted against the eccentricity, e.

Table VI: Inclination of the Orbit of the Planets to the Sun's Equatorial Plane[a]

Planet	*Longitude* (°)	B_0 (°)	☊$_0$ (°)	*e*
Mercury	58	+3.39	328	0.2056
Venus	345	+3.85	255	0.0067
Earth	345	+7.25	255	0.0167
Mars	354	+5.63	264	0.0933
Jupiter	341	+6.08	251	0.0482
Saturn	329	+5.48	239	0.0551
Uranus	346	+6.47	256	0.0480
Neptune	333	+6.39	243	0.0092
Pluto	222	+11.98	132	0.2503

[a] B_0 is the heliographic latitude, ☊$_0$ is the longitude of the ascending node with reference to the Sun's equatorial plane, and e is the eccentricity of the orbit.

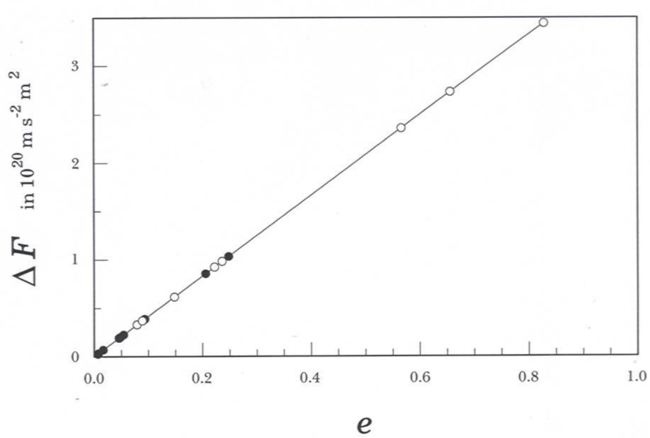

Figure 7. The sum of perturbations, ΔF, plotted against the eccentricity, e, of the orbit. Filled circles denote the orbits of the planets.

A graph of inclinations versus eccentricities is presented in Fig. 6. It can be seen that there is no correlation between the inclination and the eccentricity of the orbits of the planets. Moreover, the lowest negative heliographic latitude of the plane of the orbit of Mercury of −3.39° occurs at the ecliptic longitude of 238°; whereas the perihelion of Mercury is at the ecliptic longitude of 77.4°, about 160° apart.

We cannot expect a theory to explain the advance in the perihelion of the orbit of the planets if it cannot explain what has caused the eccentricities in the first place.

When the gravitational force is calculated by the new equation presented in Section 1 of this paper ($\mathbf{F}_S = \mathbf{a} \cdot A$), there is a highly significant correlation between the magnitude of the perturbative forces and the eccentricity of the orbit of the planets or the asteroids, as shown in Fig. 7.

The point of closest approach to the Sun, perihelion, occurs at a different angle for each planet. Figure 8 shows the longitude of perihelion, ϖ, of the planets, and the direction to the center of the Milky Way. As the planets revolve around the Sun, the Sun is also circling the center of the galaxy, and the galaxy is moving in space. Measurements of anisotropy in the cosmic microwave background radiation have detected the absolute movement of the solar system at a velocity of ~ 375 km/s in the direction of right ascension $\alpha = 11.2^h$ and declination $\delta = -7°$.[(4–6)] This is the same direction that Mercury's perihelion advances. About 0.1″ per revolution of the total advance is unaccounted for.

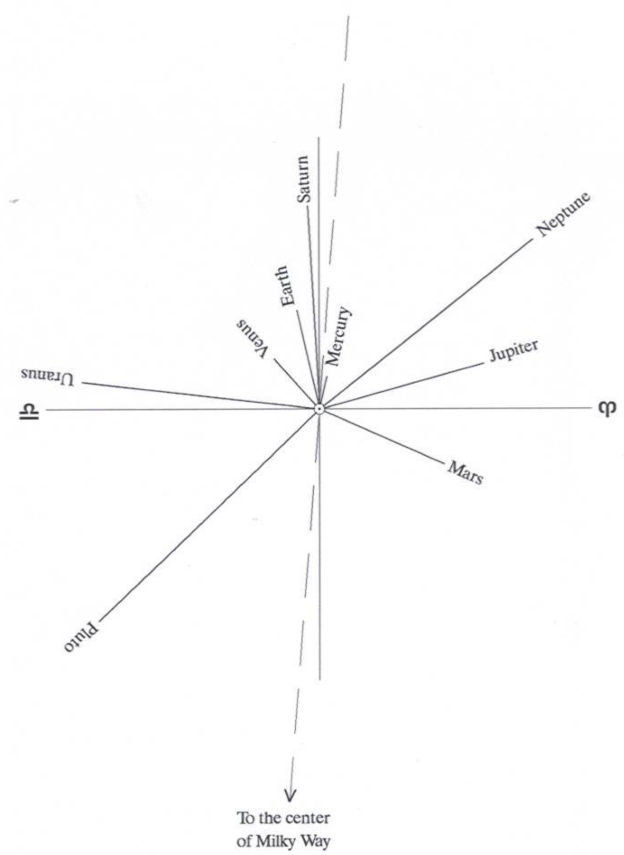

Figure 8. Longitude of the perihelion, ϖ, of planets. Distances are not to scale.

The reasons that the unaccounted advance of Mercury's perihelion is greater than that of the other planets are as follows:

a) The direction of the advance of the perihelion of Mercury coincides with th e solar system's movement in space.
b) Because Mercury's orbit is closest to the Sun, it goes through the perihelion more often than the other planets. For example, Mercury orbits the Sun a little more than four times for one revolution of the Earth, and a little more than 122 times for one revolution of Saturn.
c) Mercury's orbit has a higher eccentricity than the other planets — except Pluto; thus the effect is more noticeable.

5. THE ECCENTRICITY

The velocity of a planet at any point in an elliptic orbit around the Sun can be calculated using the equations of celestial mechanics; see, for example, Roy.[7] In extensive tables on the "Physical Data for the Planets, Their Satellites, and Some Asteroids," which were printed in the *CRC Handbook of Chemistry and Physics* from 1970 through 1985, W. Joseph Armento gives the velocities and the distances of all the planets and some asteroids at the semimajor axis of revolution, at perihelion, and at aphelion. We use his figures in the next two tables to calculate the gravitational force at semimajor, at perihelion, and at aphelion, using (4) of Section 1. We also list the eccentricities, *e*. Table VII presents data for the planets; Table VIII presents data for some asteroids.

The increment of the force at perihelion is, in all cases, equal to the negative of the increment at aphelion. The increment is due to the sum of perturbations in the direction of the line of apsides:

$$\Delta\mathbf{F} = \sum \begin{Bmatrix} \text{Perturbations in the direction} \\ \text{of the line of apsides} \end{Bmatrix}; \qquad (8)$$

$$\Delta\mathbf{F} \text{ at perihelion} = -\Delta\mathbf{F} \text{ at aphelion.}$$

Also, the sum of the forces at perihelion and at aphelion divided by two is equal to the gravitational force of the Sun:

$$\frac{\mathbf{F}\text{ at perihelion} + \mathbf{F}\text{ at aphelion}}{2} = \mathbf{F}_S. \qquad (9)$$

Referring to Fig. 9, the vector component of $\Delta\mathbf{F}$ at perihelion is in the same direction as the Sun's centripetal acceleration; $\Delta\mathbf{F}$ is added to $\mathbf{F}_S$. At aphelion the vector component of $\Delta\mathbf{F}$ is in the opposite direction to the Sun's centripetal acceleration, and the total force is $\mathbf{F}_S - \Delta\mathbf{F}$. The vector component of $\Delta\mathbf{F}$ at *S* is tangent to the curve and has no effect on the orbit of the planet. If there were no disturbing forces, the planet would orbit the Sun in a circle of radius *a*.

Going back to Fig. 7, the equation for the least squares line of regression for the nine planets, with the intervals calculated at the 95% confidence level, is

$$\Delta\mathbf{F} = (4.1657 \pm 0.0013)\cdot e \text{ in } 10^{20}\ \mathrm{m/s^2\cdot m^2}$$

or

$$e = \frac{\Delta\mathbf{F}}{\mathbf{F}_S}. \qquad (10)$$

The correlation coefficient is 0.9999.

Table VII: Gravitational Force Calculated at Semimajor, at Perihelion, and at Aphelion of the Planets

	Semimajor	*Perihelion*	*Aphelion*
Mercury: e = 0.2056			
Orbital velocity (m/s)	47 828	58 921	38 824
Distance ($\times 10^6$ m)	57 950	46 040	69 860
F ($\times 10^{20}$ m/s$^2 \cdot$ m^2)	4.1645	5.0214	3.3081
ΔF ($\times 10^{19}$ m/s$^2 \cdot$ m^2)		+8.568	–8.564
Venus: e = 0.0068			
Orbital velocity (m/s)	35 017	35 256	34 780
Distance ($\times 10^6$ m)	108 110	107 370	108 850
F ($\times 10^{20}$ m/s$^2 \cdot$ m^2)	4.1646	4.1927	4.1365
ΔF ($\times 10^{18}$ m/s$^2 \cdot$ m^2)		+2.814	–2.806
Earth: e = 0.0167			
Orbital velocity (m/s)	29 771	30 272	29 278
Distance ($\times 10^6$ m)	149 570	147 070	152 070
F ($\times 10^{20}$ m/s$^2 \cdot$ m^2)	4.1646	4.2340	4.0952
ΔF ($\times 10^{18}$ m/s$^2 \cdot$ m^2)		+6.937	–6.946
Mars: e = 0.0934			
Orbital velocity (m/s)	24 121	26 490	21 964
Distance ($\times 10^6$ m)	227 840	206 560	249 120
F ($\times 10^{20}$ m/s$^2 \cdot$ m^2)	4.1646	4.5536	3.7755
ΔF ($\times 10^{19}$ m/s$^2 \cdot$ m^2)		+3.890	–3.890
Jupiter: e = 0.0484			
Orbital velocity (m/s)	13 052	13 700	12 435
Distance ($\times 10^6$ m)	778 140	740 480	815 800
F ($\times 10^{20}$ m/s$^2 \cdot$ m^2)	4.1645	4.3662	3.9630
ΔF ($\times 10^{19}$ m/s$^2 \cdot$ m^2)		+2.017	–2.015
Saturn: e = 0.0543			
Orbital velocity (m/s)	9 638.3	10 177	9 128.4
Distance ($\times 10^6$ m)	1 427 000	1 349 500	1 504 500
F ($\times 10^{20}$ m/s$^2 \cdot$ m^2)	4.1646	4.3910	3.9385
ΔF ($\times 10^{19}$ m/s$^2 \cdot$ m^2)		+2.264	–2.261
Uranus: e = 0.0460			
Orbital velocity (m/s)	6 795.1	7 116.1	6 490.2
Distance ($\times 10^6$ m)	2 870 300	2 738 400	3 002 300
F ($\times 10^{20}$ m/s$^2 \cdot$ m^2)	4.1636	4.3563	3.9730
ΔF ($\times 10^{19}$ m/s$^2 \cdot$ m^2)		+1.926	–1.906
Neptune: e = 0.0082			
Orbital velocity (m/s)	5 427.6	5 472.3	5 383.3
Distance ($\times 10^6$ m)	4 499 900	4 463 000	4 536 800
F ($\times 10^{20}$ m/s$^2 \cdot$ m^2)	4.1646	4.1987	4.1304
ΔF ($\times 10^{18}$ m/s$^2 \cdot$ m^2)		+3.416	–3.411
Pluto: e = 0.2481			
Orbital velocity (m/s)	4 736.5	6 102.4	3 676.3
Distance ($\times 10^6$ m)	5 909 000	4 443 000	7 375 000
F ($\times 10^{20}$ m/s$^2 \cdot$ m^2)	4.1646	5.1979	3.1314
ΔF ($\times 10^{19}$ m/s$^2 \cdot$ m^2)		+10.332	–10.333

Table VIII: Gravitational Force Calculated at Semimajor, at Perihelion, and at Aphelion of Some Asteroids

	Semimajor	*Perihelion*	*Aphelion*
Ceres: e = 0.079			
Orbital velocity (m/s)	17 892	19 366	16 530
Distance ($\times 10^6$ m)	414 100	381 400	446 800
F ($\times 10^{20}$ m/s$^2 \cdot$ m^2)	4.1646	4.4938	3.8354
ΔF ($\times 10^{19}$ m/s$^2 \cdot$ m^2)		+3.292	–3.292
Pallas: e = 0.236			
Orbital velocity (m/s)	17 892	22 757	14 067
Distance ($\times 10^6$ m)	414 100	316 400	511 800
F ($\times 10^{20}$ m/s$^2 \cdot$ m^2)	4.1646	5.1477	3.1816
ΔF ($\times 10^{19}$ m/s$^2 \cdot$ m^2)		+9.831	–9.829
Vesta: e = 0.089			
Orbital velocity (m/s)	19 376	21 184	17 722
Distance ($\times 10^6$ m)	363 100	321 700	384 500
F ($\times 10^{20}$ m/s$^2 \cdot$ m^2)	4.1646	4.5354	3.7938
ΔF ($\times 10^{19}$ m/s$^2 \cdot$ m^2)		+3.708	–3.708
Eros: e = 0.222			
Orbital velocity (m/s)	24 665	30 912	19 681
Distance ($\times 10^6$ m)	217 900	169 500	266 300
F ($\times 10^{20}$ m/s$^2 \cdot$ m^2)	4.1646	5.0883	3.2405
ΔF ($\times 10^{19}$ m/s$^2 \cdot$ m^2)		+9.237	–9.240
Achilles: e = 0.148			
Orbital velocity (m/s)	13 042	15 139	11 236
Distance ($\times 10^6$ m)	779 300	664 000	894 600
F ($\times 10^{20}$ m/s$^2 \cdot$ m^2)	4.1643	4.7809	3.5481
ΔF ($\times 10^{19}$ m/s$^2 \cdot$ m^2)		+6.166	–6.162
Hidalgo: e = 0.656			
Orbital velocity (m/s)	12 372	27 146	5 639
Distance ($\times 10^6$ m)	866 000	297 900	1 434 100
F ($\times 10^{20}$ m/s$^2 \cdot$ m^2)	4.1643	6.8965	1.4326
ΔF ($\times 10^{20}$ m/s$^2 \cdot$ m^2)		+2.732	–2.732
Icarus: e = 0.828			
Orbital velocity (m/s)	28 668	93 458	8 794
Distance ($\times 10^6$ m)	161 300	27 700	294 900
F ($\times 10^{20}$ m/s$^2 \cdot$ m^2)	4.1646	7.6008	0.7165
ΔF ($\times 10^{20}$ m/s$^2 \cdot$ m^2)		+3.436	–3.448
Apollo: e = 0.566			
Orbital velocity (m/s)	24 431	46 408	12 861
Distance ($\times 10^6$ m)	222 100	96 400	347 300
F ($\times 10^{20}$ m/s$^2 \cdot$ m^2)	4.1646	6.5224	1.8047
ΔF ($\times 10^{20}$ m/s$^2 \cdot$ m^2)		+2.358	–2.360

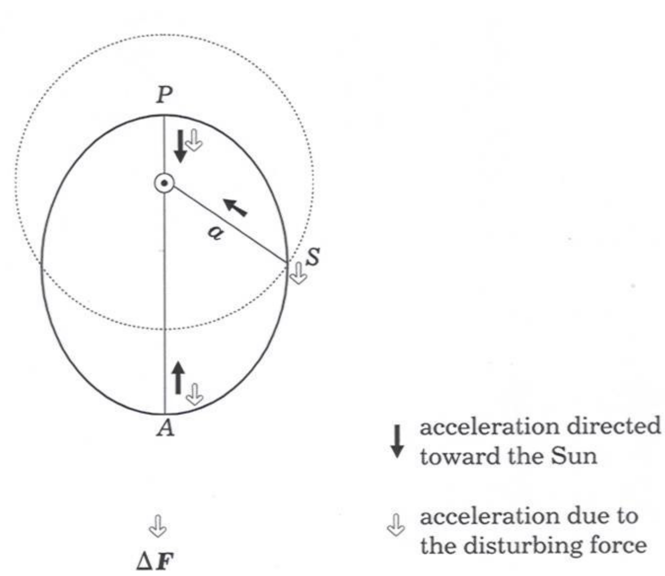

Figure 9. The effect of a perturbative force on the orbit of a planet. The Sun is at one focus. *P* is perihelion, *A* is aphelion, *S* is semimajor, and *a* is semimajor axis of revolution.

Including the asteroid data, the slope = (4.1612 ± 0.0032), with the correlation coefficient the same value of 0.9999.

Thus we see that the eccentricity is, simply, the ratio of perturbations to the gravitational force of the Sun.

6. KEPLER'S SECOND LAW

In his *Astronomia Nova* (*New Astronomy*), published in 1609, Kepler pronounced his first law of planetary motion:(8)

First Law. The orbit of a planet is elliptical, and the Sun, the source of motion, is in one of the foci of this ellipse.

This law did away with the complicated system of epicycles, deferents, and equants that was in use by astronomers prior to Kepler's time.

In his *Harmonices Mundi* (*The World Harmony*), published in 1619, Kepler pronounced his third law of planetary motion:(9)

Third Law. The periodic times of any two planets are to each other exactly as the cubes of the square roots of their mean distances.

Kepler's first and third laws were great contributions to astronomy. What is now referred to as Kepler's second law was also published in *Astronomia Nova* in 1609:(10)

Second Law. The apparent diurnal arcs of one eccentric are almost exactly proportional to the square of their distances from the Sun.

In modern terminology:

The radius vector drawn from the Sun to any planet sweeps out equal areas in equal time intervals.

The second law is now given the same status as his first and third laws, and is printed in all physics and astronomy textbooks as a general law. It may be remarked that Kepler himself has not made such a claim. After pronouncing the second law, he cautions:

Now this is true with these reservations: first, that the arcs of the eccentric be not large, that they may not have different distances varying greatly, that is, that they may not cause a sensible variation in the distances of their ends from the apsides; secondly, that the eccentricity be not very great….

Figure 10 shows a planet moving around the Sun in an elliptic orbit. In a short time Δt the radius vector r sweeps through an arc Δs. The area ΔA of the long wedge in the figure is approximately one half its base ($\approx \Delta s$) times its altitude ($\approx r$) or $r\Delta s/2$. This expression for ΔA becomes more exact in the limit as $\Delta t \to 0$. The instantaneous rate dA/dt at which area is being swept out is

$$\frac{dA}{dt} = \lim_{\Delta t \to 0} \frac{r\Delta s/2}{\Delta t} = \frac{1}{2} r \frac{ds}{dt} = \frac{1}{2} rv. \qquad (11)$$

If equal areas are swept in equal time intervals, the product of distance times velocity at any point in an elliptic orbit should be constant. We use data from tables in the *CRC Handbook of Chemistry and Physics* to calculate this product for some planets and asteroids that have orbits of high eccentricity. The subscript p denotes perihelion, the subscript a denotes aphelion, and the subscript s denotes semimajor (all values are in 10^9 m^2/s).

Mercury
$r_p v_p$ = 2 712 722.84
$r_a v_a$ = 2 712 244.64
$r_s v_s$ = 2 771 632.60
Pluto
$r_p v_p$ = 27 112 963.2
$r_a v_a$ = 27 112 712.5
$r_s v_s$ = 27 897 978.5
Hidalgo
$r_p v_p$ = 8 086 793.4
$r_a v_a$ = 8 086 889.9
$r_s v_s$ = 10 714 152.0
Icarus
$r_p v_p$ = 2 588 786.6
$r_a v_a$ = 2 593 350.6
$r_s v_s$ = 4 624 148.4

Figure 11 shows three equal areas centered at aphelion (A), at perihelion (B), and at semimajor (C). While the areas A and B are swept at approximately the same time interval, the area C is not:

$$\frac{dA}{dt} \approx \frac{dB}{dt} \neq \frac{dC}{dt}. \tag{12}$$

Kepler's second law is not a general law. Indeed, if Kepler's second law were a general law, it would be inconsistent with his first and third laws.

7. UNITS OF FORCE AND ENERGY

7.1 The Ambiguity

The confusion surrounding the concepts of *force* and *energy* has a long history in science. In the 17th century, to express force, energy, momentum, power, pressure, strength, and a number of other concepts, scientists used the word force (*forza* or *vis*). In Richard S. Westfall's words:(11)

Dynamics in the seventeenth century was bedevilled by dimensionally incompatible definitions of force.

In other words:(12)

This … was the confusion of the entire century in its jumbling together of various irreconcilable concepts under the heading of "force."

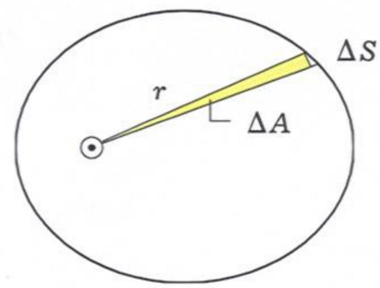

Figure 10. As a planet orbits the Sun, the radius vector r sweeps out an area ΔA in a time Δt.

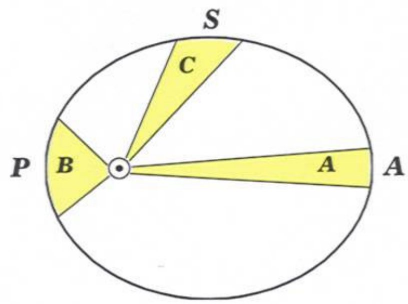

Figure 11. Three equal areas centered at aphelion, A, at perihelion, B, and at semimajor, C.

Isaac Newton did not escape the ambiguity of terms. He used the word to express more than six different kinds of physical units in his writings:(13)

Newton … defined six kinds of force — inherent force, the force of motion, exerted force, impressed force, centripetal force, and resistance. "There are also other forces," he added, "arising from the elasticity, softness, tenacity, etc., of bodies, which I do not consider here."

Although today's unit of force bears his name, Newton himself never undertook to define the term in a precise mathematical expression.(14,15)

In a letter published in 1669 the Dutch physicist Christiaan Huygens (1629–1695) noted that in elastic collisions the sum of the products of the masses and the squares of their velocities, $\sum mv^2$, was conserved.(16) Huygens called this quantity *force*.(17) In a paper published in 1686 the German philosopher and mathematician Gottfried Wilhelm Leibniz (1646–

1716) analyzed the velocity of falling bodies and concluded that the quantity of motion — measured by the product of the mass of a body and the square of its velocity, mv^2 — which he called "living force" or *vis viva*, was conserved.[18]

In a letter written in 1717 the Swiss mathematician John Bernoulli (1667–1748) used the word *energy* to describe the product of the force and the displacement.[19] The words *force* and *energy* continued to be used interchangeably by scientists for another century. The specific terms *work* and *energy* did not come into scientific use until the 19th century.

In a lecture delivered in 1805 at the Royal Institution the British physicist Thomas Young (1773–1829) argued that the Leibniz quantity of motion — mv^2 — deserved a distinct scientific name. Young coined the term *energy* for this quantity.[20] The terms *kinetic energy* and *potential energy* were introduced in the scientific literature in the middle of the 19th century.[21]

7.2 The Definitions

Weight is defined as the product of the mass of a body and its acceleration:

$$\mathbf{W} = m\mathbf{a}. \tag{13}$$

The unit of *weight* is the newton, which is the weight of a 1 kg mass at an acceleration of 1 m/s^2. 1 N ≡ 1 kg · m/s^2.

The one-dimensional, or *linear*, force is defined as the product of acceleration times the distance traveled or the displacement:

$$\mathbf{F}_l = \mathbf{a} \cdot d, \tag{14}$$

which for a constant (or nearly constant) acceleration is

$$\mathbf{F}_l = \frac{1}{2} v^2. \tag{15}$$

To avoid confusion with previously used units, one unit of linear force may be defined as one spolter sub *l*, or $\mathbf{S}_l$, equal to a force producing an acceleration of 1 m/s^2 for 1 m.

The two-dimensional, or *circular*, force is defined as the product of acceleration and area:

$$\mathbf{F}_c = \mathbf{a} \cdot A, \tag{16}$$

which for a uniform circular motion is

$$\mathbf{F}_s = \frac{v^2}{r} \cdot \pi r^2 = v^2 \cdot \pi r. \tag{17}$$

One unit of *circular force* may be defined as one spolter sub *c*, or $\mathbf{S}_c$, equal to a force producing an acceleration of 1 m/s^2 at the periphery of a circle of $1/\sqrt{\pi}$ = 0.5642 m in radius, corresponding to an area of 1 m^2.

Force is independent of mass. For example, at a given location on the surface of Earth, we may drop a 1 kg mass. Neglecting the air resistance, we observe that the object falls with an acceleration of 9.8 m/s^2. We may now drop a 2 kg mass. It will fall with the same acceleration. We may drop a 10 kg mass. Again, it will fall at 9.8 m/s^2. If we do not drop any mass at all, still a vertical acceleration of 9.8 m/s^2 exists at that location. The British physicist Robert Hooke (1635–1703) had force proportional to the square of velocity, and in 1669 he performed two experiments to prove that $\mathbf{F} \propto v^2$.[22,23]

Energy is related to effort. When we push or pull or lift a weight, we do work. The one-dimensional, or *linear*, energy is defined as the product of mass times *linear force*:

$$\mathbf{E}_l = m\mathbf{a} \cdot d, \tag{18}$$

which for a constant acceleration is

$$\mathbf{E}_l = \frac{1}{2} mv^2. \tag{19}$$

One unit of *linear energy* is one joule sub *l*, or $\mathbf{J}_l$, equal to the energy required to impart to a 1 kg mass an acceleration of 1 m/s^2 for a distance of 1 m.

The two-dimensional, or *circular*, energy is defined as the product of mass times *circular force*:

$$\mathbf{E}_c = m\mathbf{F}_c = m\mathbf{a} \cdot A, \tag{20}$$

which for a uniform circular motion is

$$\mathbf{E}_c = m\frac{v^2}{r} \cdot \pi r^2 = mv^2 \cdot \pi r. \tag{21}$$

One unit of circular energy is one joule sub *c*, or $\mathbf{J}_c$, equal to the energy required to impart to a body of 1 kg mass an acceleration of 1 m/s^2 at the periphery of a circle of $1/\sqrt{\pi}$ = 0.5642 m in radius, corresponding to an area of 1 m^2.

There are other phenomena, such as friction, pressure, momentum, etc., that I have not treated here. In general, when we have mass in an equation, we are dealing with the concept of *energy*. *Force* is independent of mass.

Acknowledgments

The author is grateful to two anonymous reviewers for comments and criticism of the manuscript.

Received 23 July 2003.

Résumé

Selon des observations, la force gravitationnelle du soleil est le produit de l'accélération et de 1'aire d'un cercle, dont le rayon est égal au demi grand axe de révolution. Cette quantité, constante pour l'ensemble des planètes, des astéroïdes et des satellites artificiels, ne dépend pas de la masse du corps attiré. L'équation de la distance moyenne séquentielle entre les planètes et le centre du soleil s'obtient ainsi : $r = B \cdot C^n$, *où B et C représentent des constantes et n le nombre séquentiel de corps. Le coefficient de corrélation est de 0,997. On en conclut que la gravitation attire par quanta. Lorsque la force gravitationnelle est calculée à l'aide de cette nouvelle équation (*$\mathbf{F}_g = \mathbf{a} \cdot A$*) une corrélation très significative existe entre la valeur absolue des forces perturbatrices et l'excentricité de l'orbite des planètes et des astéroïdes. Un graphique illustrant l'inclinaison maximale de l'orbite de chacune des planètes par rapport au plan équatorial du soleil n'indique pas de corrélation entre l'inclinaison et l'excentricité de l'orbite. Par conséquent, la relativité générale n'explique pas les excentricités. L'avance résiduelle du périhélie de la planète Mercure qui est approximativement de 0,1″ par révolution s'explique par le fait que la direction de l'avance coïncide avec la direction du mouvement du système solaire dans l'espace, comme cela a été récemment découvert en mesurant l'anisotropie du rayonnement de fond de la micro-onde cosmique. L'équation de l'excentricité est présentée comme étant le rapport de la somme des perturbations et de la force gravitationnelle du soleil. L'analyse des données a démontré que la seconde loi de Kepler n'est pas une loi générale, c'est-à-dire que les aires égales sont balayées selon des intervalles de temps plus ou moins égaux uniquement à proximité de l`aphélie et du périhélie. En effet, si la seconde loi de Kepler était une loi générale, celle-ci ne serait pas cohérente avec ses première et troisième lois. De nouvelles unités de force et d'énergie sont présentées.*

Endnotes

[1] This is a new equation for gravitational force. It replaces Newton's universal law (6) discussed in the following section and shown to be incorrect.

References

1. Urbain J.J. Le Verrier, Annales de l'Observatoire de Paris **5,** 1 (1859).
2. Simon Newcomb, *The Elements of the Four Inner Planets and the Fundamental Constants of Astronomy,* Supplement to the American Ephemeris and Nautical Almanac for 1897 (U.S. Government Printing Office, Washington, DC, 1895).
3. A. Einstein, König. Preuss. Akad. Wissenschaften (Berlin). Sitz. **2,** 831 (1915).
4. G.F. Smoot, M.V. Gorenstein, and R.A. Muller, Phys. Rev. Lett. **39,** 898 (1977).
5. Philip Lubin, Thyrso Villela, Gerald Epstein, and George Smoot, Astrophys. J. **298,** L1 (1985).
6. G.F. Smoot et al., Astrophys. J. **396,** L1 (1992).
7. A.E. Roy, *Orbital Motion,* 2nd edition (Adam Hilger Ltd., Bristol, U.K., 1982), pp. 78–79.
8. Johannes Kepler, *Harmonices Mundi* (*The World Harmony*), translated by Dr. John H. Walden (1928) in *A Source Book in Astronomy*, edited by Harlow Shapley and Helen E. Howarth (McGraw-Hill, New York, 1929), p. 35.
9. *Ibid*., p. 38.
10. *Ibid*., p. 36.
11. Richard S. Westfall, *Force in Newton's Physics: The Science of Dynamics in the Seventeenth Cen-*

tury (American Elsevier, New York, 1971), p. 437.
12. *Ibid.*, p. 136.
13. *Ibid.*, p. 451.
14. *Ibid.*, p. 452.
15. *Idem*, *The Construction of Modern Science: Mechanisms and Mechanics* (John Wiley & Sons, New York, 1971), p. 143.
16. Erwin N. Hiebert, *Historical Roots of the Principle of Conservation of Energy* (The Department of History, University of Wisconsin, Madison, WI, 1962), p. 73.
17. Ref. 1, p. 540.
18. William Francis Magie, *A Source Book in Physics* (Harvard University Press, Cambridge, MA, 1969), pp. 51–55.
19. *Ibid.*, pp. 48–50.
20. *Ibid.*, pp. 59–60.
21. Edmund T. Whittaker, *A History of the Theories of Aether & Electricity, Volume I: The Classical Theories* (Dover Publications, New York, 1989), p. 214, footnote 6.
22. Thomas Birch, *The History of the Royal Society of London for Improving of Natural Knowledge*, Vol. 2 (A. Millar, London, 1756–1757), pp. 337–339.
23. Robert Theodore Gunther, *Early Science in Oxford*, Vol. 8 (Dawsons of Pall Mall, London, 1968), pp. 186–187. See also Richard S. Westfall, British J. Hist. Sci. **3,** 245 (1967); see discussion on pp. 255–256.

Pari Spolter
Orb Publishing Company
11862 Balboa Blvd. #182
Granada Hills, California 91344-2753 U.S.A.

e-mail: orbpublishing@msn.com